

# Chromoelectric Knot in QCD

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We argue that the Skyrme theory describes the chromomagnetic (not chromoelectric) dynamics of QCD. This shows that the Skyrme theory could more properly be interpreted as an effective theory which is dual to QCD, rather than an effective theory of QCD itself. This leads us to predict the existence of a new type of topological knot, a twisted chromoelectric flux ring, in QCD which is dual to the chromomagnetic Faddeev-Niemi knot in Skyrme theory. We estimate the mass and the decay width of the lightest chromoelectric knot to be around 50 *GeV* and 117 *MeV*.

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Recently Faddeev and Niemi have conjectured the existence of a topological knot in quantum chromodynamics (QCD), a twisted chromomagnetic vortex ring which is similar to the Faddeev-Niemi knot in Skyrme theory [1, 2]. This is an interesting conjecture based on the popular view that the Skyrme theory is an effective theory of strong interaction. *The purpose of this paper is to predict the existence of a topological glueball in QCD made of the twisted chromoelectric flux ring, which is dual to Faddeev-Niemi knot in Skyrme theory. We estimate the mass of the lightest knot glueball to be around 50 GeV.* Although topological, the chromoelectric knot could be cut and decay to lowlying hadrons, due to the presence of the quarks and gluons in the theory.

The Skyrme theory has played an important role in physics, in particular in nuclear physics as a successful effective field theory of strong interaction [3, 4, 5, 6]. A remarkable feature of Skyrme theory is its rich topological structure [7]. It has been known that the theory allows (not only the original skyrmion but also) the baby skyrmion and the Faddeev-Niemi knot [2, 8]. More importantly, it contains a (singular) monopole which plays a fundamental role. In fact all the finite energy topological objects in the theory could be viewed either as dressed monopoles or as confined magnetic flux of the monopole-antimonopole pair, confined by the Meissner effect. This observation has led us to propose that the theory can be interpreted as a theory of monopoles, in which the magnetic flux of the monopole-antimonopole pairs is confined by the Meissner effect [7].

This implies that it should be interpreted as an effective theory of strong interaction which is dual to QCD, rather than an effective theory of QCD itself. This is

because in QCD it is not the monopoles but the quarks which are confined. And QCD confines the chromoelectric flux with a dual Meissner effect. This is in sharp contradiction with the popular view that the Skyrme theory is an effective theory of QCD. In the following we compare the two contrasting views, and propose a simple experiment which can tell which view is the correct one.

Let  $\omega$  and  $\hat{n}$  (with  $\hat{n}^2 = 1$ ) be the Skyrme field and the non-linear sigma field, and let

$$U = \exp\left(\frac{\omega}{2i}\vec{\sigma} \cdot \hat{n}\right) = \cos \frac{\omega}{2} - i(\vec{\sigma} \cdot \hat{n}) \sin \frac{\omega}{2},$$

$$L_\mu = U \partial_\mu U^\dagger. \quad (1)$$

With this one can write the Skyrme Lagrangian as [3]

$$\mathcal{L} = \frac{\mu^2}{4} \text{tr} L_\mu^2 + \frac{\alpha}{32} \text{tr} ([L_\mu, L_\nu])^2, \quad (2)$$

where  $\mu$  and  $\alpha$  are the coupling constants. The Lagrangian has a hidden  $U(1)$  gauge symmetry as well as a global  $SU(2)$  symmetry. With the spherically symmetric ansatz and the boundary condition

$$\omega = \omega(r), \quad \hat{n} = \hat{r},$$

$$\omega(0) = 2\pi, \quad \omega(\infty) = 0, \quad (3)$$

one has the well-known skyrmion which has a finite energy  $E \simeq 73 \sqrt{\alpha\mu}$  [3]. It carries the baryon number

$$N_s = \frac{1}{8\pi^2} \int \epsilon_{ijk} N_{ij} (\partial_k \omega) \sin^2 \frac{\omega}{2} d^3r = 1,$$

$$N_{ij} = \hat{n} \cdot (\partial_j \hat{n} \times \partial_k \hat{n}), \quad (4)$$

which represents the non-trivial homotopy  $\pi_3(S^3)$  described by  $U$  in (1). It also carries the magnetic charge

$$N_m = \frac{1}{4\pi} \int \epsilon_{ijk} N_{ij} d\sigma_k = 1, \quad (5)$$

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which represents the homotopy  $\pi_2(S^2)$  of the monopole described by  $\hat{n}$  [7].

A remarkable point of the Skyrme theory is that  $\omega = \pi$  becomes a classical solution, independent of  $\hat{n}$ . So restricting  $\omega$  to  $\pi$ , one can reduce the Skyrme Lagrangian (2) to the Skyrme-Faddeev Lagrangian

$$\mathcal{L}_{SF} = -\frac{\mu^2}{2}(\partial_\mu \hat{n})^2 - \frac{\alpha}{4}(\partial_\mu \hat{n} \times \partial_\nu \hat{n})^2, \quad (6)$$

whose equation of motion is given by

$$\begin{aligned} \hat{n} \times \partial^2 \hat{n} + \frac{\alpha}{\mu^2}(\partial_\mu N_{\mu\nu})\partial_\nu \hat{n} &= 0, \\ N_{\mu\nu} = \hat{n} \cdot (\partial_\mu \hat{n} \times \partial_\nu \hat{n}) &= \partial_\mu C_\nu - \partial_\nu C_\mu. \end{aligned} \quad (7)$$

It is this equation that allows not only the baby skyrmion and the Faddeev-Niemi knot but also the non-Abelian monopole (Notice that  $N_{\mu\nu}$  forms a closed two-form, so that it admits a potential at least locally sectionwise). This indicates that the Skyrme theory has a  $U(1)$  gauge symmetry [7]

With

$$\hat{C}_\mu = -\frac{1}{g}\hat{n} \times \partial_\mu \hat{n}, \quad (8)$$

the Lagrangian (6) can be put into a very suggestive form [7, 9],

$$\begin{aligned} \mathcal{L}_{SF} &= -\frac{\alpha}{4}\hat{H}_{\mu\nu}^2 - \frac{\mu^2}{2}\hat{C}_\mu^2, \\ \hat{H}_{\mu\nu} &= \partial_\mu \hat{C}_\nu - \partial_\nu \hat{C}_\mu + g\hat{C}_\mu \times \hat{C}_\nu. \end{aligned} \quad (9)$$

Actually with  $\sigma = \cos(\omega/2)$  the Skyrme Lagrangian (2) itself can be expressed as

$$\begin{aligned} \mathcal{L} &= -\frac{\alpha}{4}g^2(1-\sigma^2)^2\hat{H}_{\mu\nu}^2 - \frac{\mu^2}{2}g^2(1-\sigma^2)\hat{C}_\mu^2 \\ &\quad - \frac{\mu^2}{2}\frac{(\partial_\mu \sigma)^2}{1-\sigma^2} - \frac{\alpha}{4}g^2(\partial_\mu \sigma \hat{C}_\nu - \partial_\nu \sigma \hat{C}_\mu)^2 \\ &\simeq -\frac{\alpha}{4}g^2\hat{H}_{\mu\nu}^2 - \frac{\mu^2}{2}g^2\hat{C}_\mu^2 \\ &\quad - \frac{\mu^2}{2}(\partial_\mu \sigma)^2 - \frac{\alpha}{4}g^2(\partial_\mu \sigma \hat{C}_\nu - \partial_\nu \sigma \hat{C}_\mu)^2. \end{aligned} \quad (10)$$

The approximation holds for small  $\sigma$ , which describes a linearized Skyrme theory. In this expression the Skyrme theory assumes the form of a massive gauge theory (interacting with the scalar field  $\sigma$ ) in which the gauge potential is restricted by (8).

To amplify this point further, consider the  $SU(2)$  QCD for simplicity. Introducing an isotriplet unit vector field  $\hat{n}$  which selects the color charge direction (i.e., the “Abelian” direction) at each space-time point, we can decompose the gauge potential into the restricted potential  $\hat{B}_\mu$  and the gauge covariant vector field  $\vec{X}_\mu$  [10, 11],

$$\vec{A}_\mu = A_\mu \hat{n} - \frac{1}{g}\hat{n} \times \partial_\mu \hat{n} + \vec{X}_\mu = \hat{B}_\mu + \vec{X}_\mu,$$

where  $A_\mu = \hat{n} \cdot \vec{A}_\mu$  is the “electric” potential. Notice that the restricted potential is precisely the connection which leaves  $\hat{n}$  invariant under the parallel transport,

$$\hat{D}_\mu \hat{n} = \partial_\mu \hat{n} + g\hat{B}_\mu \times \hat{n} = 0. \quad (11)$$

Under the infinitesimal gauge transformation

$$\delta \hat{n} = -\vec{\alpha} \times \hat{n}, \quad \delta \vec{A}_\mu = \frac{1}{g}D_\mu \vec{\alpha}, \quad (12)$$

one has

$$\begin{aligned} \delta A_\mu &= \frac{1}{g}\hat{n} \cdot \partial_\mu \vec{\alpha}, \quad \delta \hat{B}_\mu = \frac{1}{g}\hat{D}_\mu \vec{\alpha}, \\ \delta \vec{X}_\mu &= -\vec{\alpha} \times \vec{X}_\mu. \end{aligned} \quad (13)$$

This shows that  $\hat{B}_\mu$  by itself describes an  $SU(2)$  connection which enjoys the full  $SU(2)$  gauge degrees of freedom. Furthermore  $\vec{X}_\mu$  transforms covariantly under the gauge transformation. Most importantly, the decomposition (11) is gauge-independent. Once the color direction  $\hat{n}$  is selected the decomposition follows automatically, independent of the choice of a gauge.

The advantage of the decomposition (11) is that all the topological features of the original non-Abelian gauge theory are explicitly inscribed in  $\hat{B}_\mu$ . The isolated singularities of  $\hat{n}$  defines  $\pi_2(S^2)$  which describes the Wu-Yang monopole [10, 11]. Besides, with the  $S^3$  compactification of  $R^3$ ,  $\hat{n}$  characterizes the Hopf invariant  $\pi_3(S^2) \simeq \pi_3(S^3)$  which describes the topologically distinct vacua and the instantons [9, 12]. The importance of the decomposition has recently been appreciated by many authors in studying various aspects of QCD [1, 13]. Furthermore in mathematics the decomposition plays a crucial role in studying the geometrical aspects (in particular the Deligne cohomology) of non-Abelian gauge theory [14, 15].

Notice that the restricted potential  $\hat{B}_\mu$  actually has a dual structure. Indeed the field strength made of the restricted potential is decomposed as

$$\begin{aligned} \hat{B}_{\mu\nu} &= \hat{F}_{\mu\nu} + \hat{H}_{\mu\nu} = (F_{\mu\nu} + H_{\mu\nu})\hat{n}, \\ F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu, \\ H_{\mu\nu} &= -\frac{1}{g}N_{\mu\nu} = -\frac{1}{g}(\partial_\mu C_\nu - \partial_\nu C_\mu), \end{aligned} \quad (14)$$

where now  $C_\mu$  plays the role of the “magnetic” potential [10, 11]. This shows that the gauge potential (8) which appears in the Skyrme-Faddeev Lagrangian (9) is precisely the chromomagnetic potential of QCD.

With (11) we have

$$\vec{F}_{\mu\nu} = \hat{B}_{\mu\nu} + \hat{D}_\mu \vec{X}_\nu - \hat{D}_\nu \vec{X}_\mu + g\vec{X}_\mu \times \vec{X}_\nu, \quad (15)$$

so that the Yang-Mills Lagrangian is expressed as

$$\begin{aligned} \mathcal{L}_{QCD} &= -\frac{1}{4}\hat{B}_{\mu\nu}^2 - \frac{1}{4}(\hat{D}_\mu \vec{X}_\nu - \hat{D}_\nu \vec{X}_\mu)^2 \\ &\quad - \frac{g}{2}\hat{B}_{\mu\nu} \cdot (\vec{X}_\mu \times \vec{X}_\nu) - \frac{g^2}{4}(\vec{X}_\mu \times \vec{X}_\nu)^2. \end{aligned} \quad (16)$$

This tells that QCD can be viewed as a restricted gauge theory made of the binding gluon  $\hat{B}_\mu$ , which has the valence gluon  $\vec{X}_\mu$  as a gauge covariant colored source [10, 11]. Now, suppose that the confinement mechanism generates a mass  $\mu$  for the binding gluon. Then, in the absence of  $A_\mu$  and  $\vec{X}_\mu$ , the above Lagrangian reduces exactly to the Skyrme-Faddeev Lagrangian (6). Furthermore, with

$$\begin{aligned} A_\mu &= \partial_\mu \sigma, & \vec{X}_\mu &= f_1 \partial_\mu \hat{n} + f_2 \hat{n} \times \partial_\mu \hat{n} \\ \phi &= f_1 + i f_2, & \partial_\mu \phi &= 0, \end{aligned} \quad (17)$$

we have

$$\begin{aligned} \mathcal{L}_{QCD} &\simeq -\frac{(1 - g\phi^*\phi)^2}{4} g^2 \hat{H}_{\mu\nu}^2 - \frac{\mu^2}{2} g^2 \hat{C}_\mu^2 \\ &- \frac{\mu^2}{2} (\partial_\mu \sigma)^2 - \frac{\phi^* \phi}{4} g^2 (\partial_\mu \sigma \hat{C}_\mu - \partial_\nu \sigma \hat{C}_\nu)^2. \end{aligned} \quad (18)$$

This (with  $\alpha = (1 - g\phi^*\phi)^2 = \phi^*\phi$ ) is precisely the linearized Skyrme Lagrangian in (10). So, if we like, we can actually derive the linearized Skyrme theory from QCD with simple assumptions [7]. This shows how the Skyrme theory stems from QCD. More importantly, this reveals that the Skyrme theory describes the chromomagnetic dynamics, not the chromoelectric dynamics, of QCD.

Just like the  $SU(2)$  QCD the Lagrangian (6) has the non-Abelian monopole solution [7]. It also has a magnetic vortex solution known as the baby skyrmion and a twisted vortex solution known as the helical baby skyrmion [7, 8]. The existence of the vortex solutions implies the existence of the Meissner effect in Skyrme theory. To see how the Meissner effect comes about, notice that due to the  $U(1)$  gauge symmetry the theory has a conserved current

$$j_\mu = \partial_\nu N_{\mu\nu}, \quad \partial_\mu j_\mu = 0. \quad (19)$$

Clearly this is the current which generates the Meissner effect and confines the magnetic field of the vortex [7]. This confirms that the Skyrme theory indeed has a built-in Meissner effect and confinement mechanism.

More importantly the Skyrme theory admits the Faddeev-Niemi knot, which is nothing but the twisted magnetic vortex ring made of the helical baby skyrmion [7]. It has the knot quantum number [1, 7]

$$N_k = \frac{1}{32\pi^2} \int \epsilon_{ijk} C_i N_{jk} d^3x = 1. \quad (20)$$

Obviously the knot has a topological stability. Furthermore, this topological stability is now backed up by the dynamical stability. To see this, notice that the chromoelectric supercurrent (19) has two components, the one moving along the knot and the other moving around the knot tube. And the supercurrent moving along the knot generates an angular momentum around the  $z$ -axis which provides the centrifugal force preventing the vortex ring

to collapse. Put it differently, the supercurrent generates a magnetic flux trapped in the knot disk which can not be squeezed out. And this flux provides a stabilizing repulsive force which prevent the collapse of the knot. This is how the knot acquires the dynamical stability.

One could estimate the energy of the knot. Theoretically it has been shown that the knot energy has the following bound [16]

$$c \sqrt{\alpha} \mu N^{3/4} \leq E_N \leq C \sqrt{\alpha} \mu N^{3/4}, \quad (21)$$

where  $C$  is an unknown constant equal to or larger than  $c$ . This suggests that the knot energy is proportional to  $N^{3/4}$ . Indeed numerically, one finds [17]

$$E_N \simeq 252 \sqrt{\alpha} \mu N^{3/4}, \quad (22)$$

up to  $N = 8$ . This sub-linear  $N$ -dependence of knot energy means that a knot with large  $N$  can not decay to the knots with smaller  $N$ .

Adopting the popular view that the Skyrme theory is an effective theory of QCD one can easily predict the existence of a chromomagnetic knot in QCD. Furthermore one can estimate the mass of this knot from (22). In this picture the parameters  $\mu$  and  $\alpha$  may be chosen to be [4, 5]

$$\mu = f_\pi \simeq 93 \text{ MeV}, \quad \alpha = 8\epsilon^2 \simeq 0.0442, \quad (23)$$

with the baryon mass  $m_b \simeq 1.427 \text{ GeV}$ . In a slightly different fitting one may choose [4, 6]

$$\mu = f_\pi \simeq 65 \text{ MeV}, \quad \alpha = 8\epsilon^2 \simeq 0.0336. \quad (24)$$

to have the baryon mass  $m_b \simeq 0.870 \text{ GeV}$ . So from (23) we find the mass of the lightest glueball to be

$$m_k \simeq 4.93 \text{ GeV}, \quad (25)$$

but with (24) we obtain

$$m_k \simeq 3.00 \text{ GeV}. \quad (26)$$

From this we expect the mass of the knot glueball proposed by Faddeev and Niemi to be around 3 to 5  $\text{GeV}$ .

Our result in this paper challenges this traditional view. We have shown that the Skyrme theory describes the chromomagnetic (not chromoelectric) dynamics of QCD. Moreover, the real baryon is made of quarks which carry the chromoelectric charge, while the skyrmion is actually a dressed monopole which carries the magnetic charge. And the Faddeev-Niemi knot is made of the color magnetic flux, while the glueball in QCD is supposed to carry the color electric flux. Furthermore, although our analysis implies that the Skyrme theory is a theory of confinement, what is confined here is the monopoles, not the quarks. And what confines the quarks in QCD is a dual Meissner effect, not the Meissner effect. This tells that the Skyrme theory may not be viewed as an effective

theory of QCD, but more properly as an effective theory which is dual to QCD.

This dual picture implies that QCD could admit a chromoelectric knot which is dual to the chromomagnetic Faddeev-Niemi knot. This is because one could make such a knot by twisting a  $g\bar{g}$  flux and smoothly connecting both ends. Assuming the existence one may estimate the mass of the knot. In this case one may identify  $\sqrt{\alpha}\mu$  as the QCD scale  $\Lambda_{QCD}$ , because this is the only scale we have in QCD. So, with [18]

$$\Lambda_{QCD} \simeq \sqrt{\alpha} \mu \simeq 200 \text{ MeV}, \quad (27)$$

one can easily estimate the mass of the lightest electric knot. From (22) we expect

$$M_k \simeq 50 \text{ GeV}. \quad (28)$$

The stability of such chromoelectric knot is far from guaranteed. This is because in QCD we have other fields, the quarks and gluons, which could destabilize the knot. For example, the knot can be cut and decay to  $g\bar{g}$  pairs and thus to lowlying hadrons. We could estimate the decay width of the knot from the one-loop effective action of QCD. According to the effective action the chromoelectric background is unstable and decays to  $g\bar{g}$ , with the probability  $11g^2E^2/96\pi$  per unit volume per unit time [19, 20]. So assuming that the knot is made of  $g\bar{g}$  flux ring of thickness  $1/\Lambda_{QCD}$  and radius of about  $3/\Lambda_{QCD}$ , we can estimate the decay width  $\Gamma$  of the knot

$$\Gamma \simeq \frac{11g^2}{96\pi} \left( \frac{g\Lambda_{QCD}^2}{\pi} \right)^2 \times \frac{6\pi^2}{\Lambda_{QCD}^3} \simeq 11\pi\alpha_s^2 \Lambda_{QCD}$$

$$\simeq 117 \text{ MeV}, \quad (29)$$

where we have put  $\alpha_s(M_k) \simeq 0.13$  [18]. Of course this is a rough estimate, but this implies that the chromoelectric knot can have a typical hadronic decay. In the presence of quarks, a similar knot made of a twisted  $q\bar{q}$  flux could also exist in QCD.

In this paper we have challenged the popular view of Skyrme theory, and provided an alternative view. There is a simple way to determine which is the correct view. This is because the two views predict totally different knot glueballs which could be verified by the experiments. We have argued that the knot in traditional view is a chromomagnetic knot, while the knot we predict here is a chromoelectric knot. More importantly, we have shown that in the traditional view the mass of the lightest knot glueball should be around 3 to 5 GeV, but in the dual picture the mass of such glueball should be around 50 GeV. So, experimentally one could tell which is the correct view simply by measuring the mass of the exotic knot glueball. Certainly the LHC could be an ideal place to determine which view is correct. The details of our argument will be published elsewhere [21].

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